CALCULUS 2

Below are some foundational equations for Calculus 2, for your review.

Derivatives

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = \sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(x) = (n-1)x^{n-1}$$

$$\frac{d}{dx}(e^t) = x'e^x$$

Integrations

$$\int x\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \cos x\,dx = \sin x + c$$

$$\int \sin x\,dx = \cos x + c$$

$$\int \tan x\,dx = \ln|\sin x| + c$$

$$\int e^x\,dx = e^x + c$$

Limits

$$\lim_{n\to a} f(x) = L$$

(Discontinuity)

$$\lim_{n\to a^x} f(x) = A$$

$$\lim_{n\to a^-} = B$$

$$\lim_{n\to a^-} = \text{ONE}$$

$$\lim_{n\to a} f(x) \neq$$

(Limits at ∞)

$$\lim_{x\to\infty} \frac{a}{x} = 0$$

$$\lim_{x\to\infty} \frac{ax^n}{bx^n} = \frac{a}{b}$$

$$\lim_{x\to\infty} \frac{ax^m}{bx^n} = \text{if } m > n$$

$$\lim_{x\to\infty} \frac{ax^m}{bx^n} = \text{if } n > m + 0$$

Under / Area Between Curves

$$A = \int_a^b f(x)\,dx$$

$$A = \int f(x) - g(x)\,dx$$

$$A = \int_c^d f(y) - g(y)\,dy$$

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Math 157 **Calculus II** **Final Exam** **12/6/2016**

SHOW ALL WORK. Justify your answers!
Simplify your answers. Give exact answers whenever possible.
Each problem = 20 pts. Solve any 10 problems below. Exam total = 200 pts.

1. Find the area bounded by the curves $y = x$, $y = \tan x$, and $x = \pi/4$.

2. Find the coordinates of the centroid of the region bounded by $y = \cos x$, $y = \sin x$, $x = 0$, $x = \pi/4$.

3. Express the integrand as a sum of partial fractions and then evaluate the integral
$$\int \frac{x+4}{x^3+4x}\,dx$$

4. Evaluate the following integrals:
   (a) $\displaystyle\int_0^{\pi/2} \cos^3 x \sin^2 x\,dx$
   (b) $\displaystyle\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

5. Evaluate the following integrals:
   (a) $\displaystyle\int \frac{dx}{\sqrt{2x-x^2}}$
   (b) $\displaystyle\int \cos(\ln x)\,dx$ *Hint:* First make a substitution, then use integration by parts.

6. Find the general term a_n of the series
$$1 + \frac{1\cdot 2}{1\cdot 3} + \frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5} + \frac{1\cdot 2\cdot 3\cdot 4}{1\cdot 3\cdot 5\cdot 7} + \frac{1\cdot 2\cdot 3\cdot 4\cdot 5}{1\cdot 3\cdot 5\cdot 7\cdot 9} + \cdots$$
   and determine whether the series converges.

7. Determine whether the series are absolutely convergent, conditionally convergent, or divergent (and give reasons for your answers):
   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n\,n!}{n^n}$

8. Find the Taylor series for the function $f(x) = x^3 - 3x$ about $x = 1$.

9. Find the Maclaurin series representation of $f(x) = \tan^{-1} x$.

10. Find the radius of convergence and the interval of convergence (including endpoints where needed) of the series

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n3^n}$$

11. (a) Find the length of curve $y = \cosh x$ for $-1 \leq x \leq 1$.

    (b) Find the area of the surface obtained when the infinite curve $y = e^{-x}$, $x \geq 0$, is rotated about the x-axis.

12. Find the resulting volume when the region between $y = e^{-x^2}$ and $x = 0$ and to the right of $y = 0$ is rotated about the y-axis.

13. Find the foci and eccentricity of the ellipse given by $\dfrac{(x+2)^2}{225} + \dfrac{(y-1)^2}{81} = 1$.

14. Determine if the sequence (a_n), where $a_n = \dfrac{(\ln n)^2}{n}$, converges or diverges. If it converges, compute the limit.

15. Find the area bounded by the curve $r = 4(1 + \sin\theta)$ in polar coordinates.

16. Find the slope of the tangent line to $r = 4(1 + \sin\theta)$ at $\theta = \pi/4$.